Title: Investigation of Portfolio Optimization with Realistic Constraints in Indian Equity Market Using Random Immigration Genetic Algorithm

Abstract:

Optimal allocation of limited assets for maximization of the return and minimization of the risk is a challenge faced by rational investors. Markowitz's MV model helps in formulating the problem quantitatively. However, the researchers criticized the MV model due to its' distance from reality. In this thesis, we critically evaluated the assumptions and the real-life constraints the investors face. Subsequently, we formulated a realistic MV model. With the addition of the constraints, the problem becomes more complex to be solved by classical methods. So following other researchers, we explored the Genetic Algorithm (GA), an evolutionary algorithm. But, as the GA suffers from the problem of balance between exploration and exploitation, we propose two methods to balance the same, i.e., Harmonic Progression NSGA-II (HPNSGA-II), and Random Immigration NSGA-II (RINSGA-II). Experimental results suggest that the performance of the HPNSGA-II algorithm is marginally worse than the NSGA-II algorithm when applied to Portfolio Optimization Problem. However, we found that the RINSGA-II algorithm is generating better spread compare to NSGA-II. Later on, we applied the constraints gradually and checked the efficacy of the algorithm. Subsequently conducted the out of sample test.

Introduction:

The presence of multiple conflicting objectives like maximization of return and minimization of the risk and numerous numbers of investable assets, make portfolio optimization a challenging problem.

Markowitz (1952) proposed a quantitative framework of the mean-variance (MV) model to achieve the goal of portfolio optimization by maximizing the return against a given level of risk or minimizing risk against a given level of return. It further elaborates on the role of diversification, the interrelationship among different assets and the overall impact of these interplays in reducing the risk of the portfolio.

However, the MV framework, due to the assumptions, is subjected to widespread criticism. First, that assets' returns are assumed to be Gaussian distributed; in reality, it is found leptokurtic (Mills, 1997), assumption of infinite divisibility of an asset is impractical; the absence of tax or transaction costs is far from reality.

Besides, it uses only two constraints, i.e., budget and no short sales. However, in reality, investors are subjected to a lot more constraints. The inclusion of these constraints makes the problem complicated to solve using classical optimization techniques. Hence, evolutionary algorithms like Genetic Algorithms (GA), Ant colony optimizations are explored.

However, when NSGA-II, a popular algorithm of the GA family, applied to the MV model, it was noticed that the NSGA-II fails to trace the entire Pareto optimal front, especially the high-risk area.

To address these gaps, this study checks the effectiveness of using the GA, a metaheuristic algorithm, to construct an optimized portfolio with a higher number of realistic constraints.

Further, we proposed two modifications to GA and assessed the effectiveness of these modifications in tracing the efficient frontier. The observation is that the modified algorithm facilitates to extend to the broader area of Pareto front including the high-risk area.

Literature Review:

Researchers criticize the MV model extensively. The model's assumptions of zero transaction cost or taxes and the infinite divisibility of market instruments do not reflect market reality. Consequently, researchers have tried to extend this model by incorporating multiple constraints like a) Cardinality constraints, (Busetti, 2000; Chang et al., 2000; Schaerf, 2002; Crama&Schyns, 2003; Skolpadungket, 2007; Chiam et al., 2008; Soleimani et al., 2009; Anagnostopoulos &Mamanis, 2010), b) Floor and ceiling constraints, (Crama&Schyns, 2003; Streichert et al., 2004; Streichert et al., 2004a; Fernández & Gómez, 2007); c) Round lot constraints, (Skolpadungket, 2007; Streichert et al., 2004a; Streichert et al., 2004b), etc.

Markowitz's original mean-variance model is:

$$Minimize\ Variance_{portfolio} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \quad (1)$$

Subject to

$$Expected\ Return_{Portfolio} = \sum_{i=1}^{n} w_i \mu_i \quad (2)$$

$$\sum_{i=1}^{n} w_i = 1$$

where $i = 1, 2, \ldots n$ and $w_i \geq 0$ (3)

where

σij = covariance between the two assets i and j ($i = 1, 2\ldots, n; j = 1, 2\ldots\ldots, n$)

μ_i= Expected Return of assets i (i = 1, 2… n)

wi= percentage of investable fund allocated to a particular asset

The critical assumptions are:

a) *Investors are rational*

b) Investors are risk-averse

c) The Short sale is not allowed

d) Omitted taxes and transaction costs

e) Assets returns are normally distributed

f) Securities can be divided into fractions and traded

Major Criticism of Mean-variance Model

MV came under criticism as:

a) Returns are not Multivariate Normal
b) A large number of Variance-covariance calculations
c) Risk/Return estimation using historical prices may suffer from estimation error leading to either overestimation or underestimation of predicted values. These estimation errors may impact the expected returns during portfolio building (Michaud, 1998).
d) Impractical assets weight allocation
e) Variance as Risk Measure

Real-World constraints applied on Portfolio in Literature

In MV, Markowitz (1952) has implemented only two constraints. However, in reality, investors face many more realistic constraints arising from pre-specified investment mandates, business/industrial regulations, and other practical aspects (Skolpadungket 2007). So, researchers use different types of constraints. Metaxiotis and Ligkouras (2012) elaborated that widely used constraints are:

a) Cardinality.
b) Floor-ceiling.
c) Transaction round-lots.
d) Sector capitalization.
e) Transaction cost.
f) Non-negativity, and
g) 5-10-40 constraint.

However, the majority of the researchers have used a maximum of two constraints to optimize the portfolio using a multi-objective evolutionary algorithm (Metaxiotis and Liagkouras 2012). Therefore, there leaves the possibility of exploring a higher number of constraints while optimizing the portfolio.

Classification of the constraints

In the current study, we extended the classification by Kolm et al. (2014) like

Regulatory Constraints

To create a level playing ground for all investors, the market regulator, imposes certain restrictions like:

1. In India, as per the SEBI (The Securities and Exchange Board of India) Act, 1992, Investment advisers are barred from advising the securities of their own company.
2. The executives of a company are discouraged from trading the securities.
3. 5-10-40 constraint, as per German Investment Law (2005), applicable for the mutual fund's industry. The rule states that "*a maximum of 10 percent of a fund's net assets may be invested in securities from a single issuer, and that investments of more than 5 percent with a single issuer may not make up more than 40 percent of the whole portfolio.*" (Alfi, 2015).

Branke et al. (2009) applied additional constraints like cardinality constraints and 5-10-40 constraint in portfolio construction.

Guideline Constraints

Imposed by the Investor, like:

1. Invest only in equities that are a part of the benchmark, like Nifty 50, SENSEX.

2. Cardinality constraint, one of the most used constraints, by the researchers, (Busetti, 2000; Chang, Meade, and Sharaiha 2000; Schaerf, 2002; Crama and Schyns, 2003; Skolpadungket, 2007; Chiam, Tan, and Mamum, 2008; Soleimani, Golmakani, and Salimi 2009; Anagnostopoulos and Mamanis, 2010; Golmakani and Frazel, 2011).

Cardinality constraint represented as:

$C_l \leq \sum_{k=1}^{N} z_k \leq C_u and\ 0 \leq C_l \leq C_u \leq N$

where

Cl, Cu = Lower and Upper limit respectively, i.e., the minimum, maximum number of equities in the Portfolio

z = binary value of 0 or 1 to indicate the presence or absence of an equity

N = the maximum number of equities

3. Instructions not to invest in a specific sector due to experience or social consciousness.

4. No short selling is used in the original model but was later relaxed by Black, Jensen, and Scholes (1972)

Trading Constraints

1. Imposed by the market like trading of a fraction of security is not permissible. To adhere to this researchers have applied round-lot constraint (Lin, Wang and Yan, 2001; Streichert, Ulmer and Zell, 2004; Streichert, Ulmer and Zell, 2004a; Gomez, Flores and Osorio, 2006; Skolpadungket, 2007; Soleimani, Golmakani, and Salimi, 2009; Li and Xu, 2013; Meghwani and Thakur, 2017) like:

$w_iC = L_iP_i$

where w_i = weight of the asset, C = total investable capital, L_i = tradable lot size of the assets and Pi = Price of the Lot

2. Liquidity constraint arises due to limited tradability of the asset.

3. Buy-in threshold constraint corresponds to trading in low numbers, which impacts the returns, considering transaction costs. The mathematical formulation of the constraints is:

$$a_i \leq w_i \leq b_i, 0 \leq a_i \leq b_i \leq 1$$

where

a_i, bi = Floor and Ceiling constraint on weights,

w_i = weight in the portfolio

Streichert, Ulmer and Zell, (2004), Skolpadungket (2007), Gomez, Flores, and Osorio, (2006), Li and Xu, (2013) have used floor constraint. However, both floor and ceiling constraints imposed extensively (Crama and Schyns, 2003; Hochreiter 2007; Hochreiter, 2008).

4. Transaction cost plays a significant role, specifically in the portfolio rebalancing scenario, in the multi-period case.

Maringer et al. (2005) presented this as:

$$C_i = \begin{cases} C_f & (Fixed\ cost\ only) \\ c_p.n_i.S_{0i} & (Proportional\ cost\ only) \\ max\{C_f, c_p.n_i.S_{0i}\} & (Proportional\ cost\ with\ lower\ limit) \\ C_f + c_p.n_i.S_{0i} & (Proportional\ cost + fixed\ cost) \end{cases}$$

where,

n_i =Natural, positive numbers of assets i={1, … …. . , N}

S_{0i} =Present price of the asset

C_f = Fixed cost

c_p = Proportional cost

Some of the past studies used transaction cost like Chen et al. (2015), Silva et al. (2015).

Risk Management Constraints

Generally, a portfolio manager imposes constraints on her choices to limit exposure like:

1. Ceiling constraint helps to reduce the effect of the estimation errors as the exposure of the portfolio to a particular asset can be limited (Gupta et al. 1998).

Researchers have used this constraint along with floor constraints, like, Chang, Meade, and Sharaiha (2000), Crama and Schyns (2003), Pai and Michel (2009), Soleimani, Golmakani and Salimi (2009).

2. To reduce the exposure to a particular sector, the sectoral constraint was added to the formulation of portfolio optimization (Soleimani, Golmakani, and Salimi 2009, Anagnostopoulos and Mamanis, 2010).

Discretionary Constraints

A portfolio manager can exercise other discretionary constraints like:

a) *Adequate Size of the Enterprise*

b) *Sufficiently Strong Financial Condition*

c) *Earnings Stability*

d) *Dividend Record*

e) *Earnings Growth*

Research Gap

Most of the researchers used only 2-3 additional constraints compared to the MV model. So to make the model more realistic, we are proposing to use the following constraints:

- **Guideline Constraints**
  - ✓ Budget

- ✓ No short sale
- ✓ Cardinality (range)

- **Trading Constraints**
  - ✓ Round lot
  - ✓ Liquidity
- **Risk Management Constraints**
  - ✓ Floor/Buy in Threshold and Ceiling
  - ✓ Sectoral

Additionally, Transaction Cost

Proposed Model for consideration

$MinimizeVariance_{portfolio} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$ (1)

$Maximize\ ExpectedReturn_{Portfolio} = \sum_{i=1}^{n} w_i \mu_i$ (2)

Subject to

$\sum_{i=1}^{n} w_i \leq 1$, i =1,2,.....n(Budget Constraints) (A)

$0 \leq w_i \leq 1$ (Non-negativity Constraints) (B)

$C_l \leq \sum_{k=1}^{n} z_k \leq C_u$ $,z = \{0,1\}$ (Cardinality Constraints) (C)

$0 \leq C_l \leq C_u \leq n$

$a_i \leq w_i \leq b_i$(Floor and Ceiling constraints) (D)

$0 \leq a_i \leq b_i \leq 1$

$a_i \geq \max\ (userdefinedlowerlimit, \frac{100XC_f}{w_i CC_p})$

Ct=max $\{Cf, p.ni.Soi\}$ $(Proportionaltransactioncost)$ (E)

$w_iC = L_iP_i$ (Round Lot Constraints) (F)

$S_l \leq w_iC \leq S_u$ (Sectoral Constraints) (G)

$w_iC \leq AverageTradingvolume$ (Liquidity Constraints) (H)

Where

w_i = weight, i.e., percentage of fund allocated to a particular asset

σ_{ij} = covariance between the two assets i and j (i =1,2,….,n; j = 1,2,…..,n)

μ_i= Expected Return of assets i (i = 1, 2… n)

n = maximum number of assets

C_l, C_u = Lower, upper limit, i.e., the minimum, maximum number of assets to be part of the Portfolio

a_i, bi = lower limit or floor constraint and upper limit or ceiling constraint on weights,

C = total investable capital

L_i = tradable lot size of the assets

P_i = Price of the Lot

Z = Assets or individual equity

S_l, S_u= Lower, upper limit in a sector

n_i =Natural, positive numbers of assets i={1, … …. . , N}

S_{0i} = Present price of the asset

C_f = Fixed cost

c_p = Proportional cost

The necessity of other Algorithms

The inclusion of more factors makes the problem of portfolio optimization intractable. Hence, Researchers in the past have extensively used heuristics like Simulated Annealing (Crama and Schyns 2003), Particle Swarm optimization (Zhang and Liu 2017), Tabu Search (Chang, Meade and Sharaiha 2000), Fireworks Algorithm (Zhang and Liu 2017). Following the same, we have used the Genetic Algorithm to solve the enhanced portfolio optimization problem.

Methodology and Empirical Results

Optimization is a search process to explore the variable space for obtaining a combination, which generates optimal solutions. However, as posited by Herrera and Lozanzo (1996), "*The genetic algorithm behavior is determined by the exploitation and exploration relationship kept throughout the run*." Too much exploration will prevent the algorithm from reaching the Pareto-optimal solution whereas, too little exploration will not help in discovering the diverse solutions, especially in the case of the multi-objective optimization process.

Here, we are proposing the two methods of diversity maintenance:

1) Controlled elitism in NSGA-II using Harmonic Progression and
2) Infusion techniques in a static environment using a specific initialization mechanism

Harmonic Progression Distribution

In the harmonic distribution, the non-dominated front gets the highest slots, and the subsequent front gets smaller slots. The formulation is:

$$N_i = N * \frac{1}{apStartRate + (i-1) * aprate} * \frac{1}{\sum_{j=1}^{k}(apStartRate + (j-1) * aprate)}$$

Where

N_i = number of allowed solutions in i^{th} set

i , j = set number

k = total number of sets

used apStartRate= 1.0 and aprate = 1.0

When the number of slots is more than the number of solutions in the available front, the slots available to the second front will increase by the number of empty slots in the previous front. This process will continue until the last fronts.

When the solutions in the front are more than the number of slots available in the front, the solution set having the best crowding distances selected;

By controlling the non-dominated front population, we increase the exploration capability and reduce the exploitation capability to bring a balance between the two.

Infusion Techniques / Random Immigration

We introduced the concept of random immigration (RI) (RINSGA-II) to enhance exploration in a static environment. In the past, the random immigration concept was used in case of dynamic optimization problems (Tin´os and Yang, 2007).

RINSGA-II algorithm introduces new populations, with one gene populated, in every cycle by randomly discarding the existing ones from the current population after mutation.

The generic steps of the proposed modification are:

Step 1: Set the replacement rate (the recommended value is between 0.2 – 0.3).

Step 2: Until the required number of solutions are replaced, go through the below steps

Step 2-1: Select a random solution (chromosome) from the old population.

Step 2-2: Generate a new solution with all the weights initialized with zero

Step 2-3: Select a random position, i.e., gene within the solution

Step 2-4: Populate the solution's randomly selected gene with a random value generated between the lower and upper limits. Other weights remain zero.

Step 2-5: Replace the old solution with the new solution in the population.

1.1. Experimental Setup

To evaluate, we first applied the ZDT1, ZDT2, ZDT3, and ZDT6 of Zitzler-Deb-Thiele (Zitzler, Deb, and Thiele, 2000) and Tanaka (1995).

Subsequently, we applied it to the Portfolio Optimization Problem sets, published in the OR-Library (Beasley 2019). The five sets of data have a different number of equities ranged from N=31 (Hang Seng) to N =225 (Nikkie) (Chang, Meade and Sharaiha, 2000).

Table 1: Portfolio details of OR Library

Portfolio Set	No of Equity
Portfolio 1	31
Portfolio 2	85
Portfolio 3	89
Portfolio 4	98
Portfolio 5	225

We have introduced the modification in the MOEA Framework version 2.12, open-source Java framework, for implementing a Multi-objective Evolutionary Algorithm by David Hadka (2017), as three separate algorithms. To measure the performance we have used: Generational Distance (GD), Hypervolume (HP) and Spacing

Table 2: Runtime Parameters sets

Parameters	NSGA-II/ GPNSGA-II/HPNSGA-II	RINSGA-II
Population Size	100	100
Encoding	Real	Real
Type of Cross over	Simulated Binary Cross over	Simulated Binary Cross over
Probability	1.0	1.0
Distribution Index	25.0	25.0
Type of Mutation	Polynomial Mutation	Polynomial Mutation
Probability	1/number of Variables	1/number of Variables
Distribution Index	30.0	30.0
No of Iteration	500	500
Random Immigration Rate	0	0.3

1.2. Discussion of Empirical Result

The results indicate having no significant difference from the performance point of view of the algorithms' results. The results obtained for ZDT1 are:

ZDT1

NSGA-II:
Hypervolume:
Min: 0.6597835881732712
Median: 0.6605606625532705
Max: 0.6610212433993744
Count: 50
Indifferent: [ReplacementNSGA-II]*

* Indicates that there is no significant difference ($p < 0.05$) between the results achieved by NSGA-II and ReplacementNSGA-II as far as Hypervolume is concerned.

GenerationalDistance:
Min: 4.356072869042689E-5
Median: 1.300633965393269E-4
Max: 2.4439583310829514E-4
Count: 50
Indifferent: [HPNSGA-II, GPNSGA-II, ReplacementNSGA-II]#

# Indicates that there is no significant difference ($p < 0.05$) between the results achieved by NSGA-II, ReplacementNSGA-II, HPNSGA-II, and GPNSGA-II as far as Generational Distance is concerned.

Spacing:

Min: 0.005526677129194123
Median: 0.0067776460991925205
Max: 0.008244005172307271
Count: 50
Indifferent: [HPNSGA-II, GPNSGA-II, ReplacementNSGA-II]^

^ Indicates that there is no significant difference ($p < 0.05$) between the results achieved by NSGA-II, ReplacementNSGA-II, HPNSGA-II, and GPNSGA-II as far as Spacing is concerned.

HPNSGA-II:
Hypervolume:
Min: 0.6596980209632695
Median: 0.6603635953173148
Max: 0.660905603511518
Count: 50
Indifferent: [GPNSGA-II]

GenerationalDistance:
Min: 6.643537954288555E-5
Median: 1.3006174680656293E-4
Max: 2.8055933697808643E-4
Count: 50
Indifferent: [NSGA-II, GPNSGA-II, ReplacementNSGA-II]

Spacing:
Min: 0.005246892385049411
Median: 0.006886630336468519
Max: 0.008724144509542306
Count: 50
Indifferent: [NSGA-II, GPNSGA-II, ReplacementNSGA-II]

Independent sample t-test indicates that from the hypervolume measurement perspective, GP NSGA-II and HP NSGA-II are statistically indifferent, whereas NSGA-II and RINSGA-II are statistically indifferent.

Table 3: Performance Measure - NSGA-II, HPNSGA-II, GPNSGA-II, and RINSGA-II of ZDT2, ZDT3, ZDT6, and Tanaka

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
ZDT2-NSGA-II	0.3270	0.3274	0.3281	3.32E-5	9.56E-5	1.71E-5	0.0060	0.007	0.009
ZDT2-HPNSGA-II	0.3267	0.3273	0.3278	6.72E-5	1.29E-4	2.67E-4	0.0055	0.007	0.009
ZDT2-GPNSGA-II	0.3267	0.3272	0.3277	5.20E-5	1.23E-4	1.94E-4	0.0055	0.007	0.009
ZDT2-RINSGA-II	0.3267	0.3274	0.3277	5.81E-5	1.18E-4	1.92E-4	0.0060	0.007	0.009
ZDT3-NSGA-II	0.5151	0.5134	0.5157	3.62E-5	5.62E-5	1.28E-5	0.0061	0.007	0.009
ZDT3-HPNSGA-II	0.5151	0.5134	0.5155	2.48E-5	5.91E-4	1.37E-4	0.0052	0.008	0.009
ZDT3-GPNSGA-II	0.4744	0.5154	0.5155	4.12E-5	6.07E-5	0.005	0.0058	0.008	0.009
ZDT3-RINSGA-II	0.5152	0.5154	0.5156	2.87E-5	5.81E-5	1.26E-5	0.0061	0.008	0.010
ZDT6-NSGA-II	0.4002	0.4011	0.4020	9.53E-5	1.54E-4	2.13E-4	0.0038	0.005	0.007
ZDT6-HPNSGA-II	0.4006	0.4015	0.4020	8.94E-5	1.27E-4	1.99E-4	0.0045	0.0056	0.0071

ZDT6-GPNSGA-II	0.4001	0.4014	0.4022	8.80E-5	1.31E-4	0.010	0.0045	0.0056	0.0885
ZDT6-RINSGA-II	0.4028	0.4032	0.4034	0.00	1.13E-5	0.0013	0.0043	0.0056	0.0126
Tanaka-NSGA-II	0.3049	0.3058	0.3060	8.24E-5	1.20E-4	1.54E-4	0.0030	0.0041	0.0049
Tanaka-HPNSGA-II	0.3054	0.3058	0.3060	7.00E-5	1.21E-4	1.59E-4	0.0032	0.0041	0.0050
Tanaka-GPNSGA-II	0.3054	0.3058	0.3063	6.94E-5	1.13E-4	2.01E-4	0.0030	0.0041	0.0053
Tanaka-RINSGA-II	0.3054	0.3056	0.3060	7.99E-5	1.30E-4	1.68E-4	0.0031	0.0041	0.0056

Number of runs: 50

HV: Hypervolume

GD: Generational Distance

MIN, MED, MAX: Minimum, Median, Maximum value obtained in the total number of runs

OR-Library Portfolios

MV model without Cardinality Constraints Using NSGA-II, HPNSGA-II, and GPNSGA-II:

To explore the efficacy of the algorithms concerning Portfolio optimization problems, we compare the results of the GPNSGA-II, HPNSGA-II, and NSGA-II, by applying on the unconstrained Portfolio Optimization Problem sets published in the OR-Library (Beasley, 2019). The results obtained, as far as the performance measures HV, GD, and Spacing are concerned, are depicted in Table 4:

Table 4: Performance Measure - NSGA-II, HPNSGA-II, GPNSGA-II, and RINSGA-II

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Portfolio-1- NSGA-II	0.6769	0.7289	0.7515	4.26E-4	5.36E-4	6.75E-4	2.20E-5	3.04E-5	5.01E-5
Portfolio-1-HPNSGA-II	0.6272	0.6798	0.7290	8. 90E-4	0.0012	0.0016	2.91E-5	3.69E-5	5.38E-5
Portfolio-1-GPNSGA-II	0.6281	0.6748	0.6946	0.0049	0.0061	0.0078	6.23E-5	0.0061	0.0078
Portfolio-2-NSGA-II	0.5829	0.6531	0.6940	6.11E-4	8.10E-4	0.0013	1.15E-5	1.42E-5	2.37E-5
Portfolio-2-HPNSGA-II	0.5405	0.6065	0.6683	0.0011	0.0014	0.0022	1.33E-5	1.87E-5	2.65E-5
Portfolio-2-GPNSGA-II	0.5605	0.5983	0.6433	0.0043	0.0061	0.0080	3.47E-5	4.67E-5	7.83E-5
Portfolio-3-NSGA-II	0.5616	0.6550	0.7062	5.59E-4	8.56E-4	0.0013	1.05E-5	1.40E-5	2.23E-5
Portfolio-3-HPNSGA-II	0.5388	0.5894	0.6547	5.26E-4	7.55E-4	0.0013	8.73E-6	1.25E-5	2.38E-5
Portfolio-3-GPNSGA-II	0.5238	0.5900	0.6560	5.30E-4	7.24E-4	0.0011	8.75E-6	1.19E-5	1.86E-5

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Portfolio-4-NSGA-II	0.6712	0.7135	0.7848	5.86E-4	7.75E-4	0.0012	1.64E-5	2.09E-5	2.73E-5
Portfolio-4-HPNSGA-II	0.6054	0.6770	0.7469	8.95E-4	0.0013	0.0019	1.99E-5	2.60E-5	4.14E-5
Portfolio-4-GPNSGA-II	0.5968	0.6766	0.7234	0.0038	0.0050	0.0071	4.22E-5	6.15E-5	9.56E-5
Portfolio-5-NSGA-II	0.6959	0.7381	0.7668	6.43E-4	9.38E-4	0.0014	7.13E-6	8.93E-6	2.09E-5
Portfolio-5-HPNSGA-II	0.6722	0.7151	0.7594	0.0016	0.0022	0.0030	9.51E-6	1.41E-5	2.27E-5
Portfolio-5-GPNSGA-II	0.6461	0.6889	0.7112	0.0063	0.0084	0.0012	2.43E-5	3.73E-5	6.89E-5

As indicated above, the value of hypervolume (HV) is more for NGSA-II than GPNSGA-II or HPNSGA-II. Hence, it shows that for high mutation rate NSGA-II produces better results concerning HV.

With respect to Generational Distance (GD), the results indicate better performance for NSGA-II. However, the results for HPNSGA-II is better than GPNSGA-II.

The independent-sample t-test conducted on HV values for HPNGSA-II (*M=0.6815, SD=0.0194*) and GPNSGA-II (*M=0.6727, SD=0.0139*) indicate that there is no significant difference in the HV values of GPNGSA-II and HPNSGA-II p (<0.05) = 0.011. A T-test on HV values for NSGA-II (M=0.72, SD=0.0017) and HPNGSA-II (*M=0.6815, SD=0.019*) indicates better convergence to spread and a significant difference in the HV generated by NGSA-II and HPNSGA-II p (<0.05) = 0.00.

MV model without Cardinality Constraints Using NSGA-II, RINSGA-II:

Figure:1 displays the Pareto front produced by both these algorithms on five portfolios, along with the results published in the OR-Library. We found that both the algorithms converge to the Pareto; however, spacing in RINSGA-II is extensive than that of NSGA-II. Table:4 presents the details.

Table 5: Performance Measure -NSGA-II and RINSGA-II

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Portfolio-1-NSGA-II	0.6281	0.6805	0.7200	2.74E-4	4.73E-4	6.32E-4	2.15E-5	2.68E-5	4.73E-5
Portfolio-1-RINSGA-II	0.7651	0.7661	0.7667	2. 21E-4	2.93E-4	3.55E-4	3.39E-5	4.08E-5	5.21E-5
Portfolio-2-NSGA-II	0.6113	0.6868	0.7522	3.96E-4	5.81E-4	0.0013	1.53E-5	2.17E-5	3.01E-5
Portfolio-2-RINSGA-II	0.8863	0.8874	0.8880	2.32E-4	2.79E-4	3.53E-4	3.05E-5	3.93E-5	4.94E-5
Portfolio-3-NSGA-II	0.5792	0.6452	0.7124	6.33E-4	9.09E-4	0.001	1.13E-5	1.57E-5	2.03E-5
Portfolio-3-RINSGA-II	0.7878	0.7899	0.7916	3. 73E-4	4.64E-4	5.45E-4	1.76E-5	2.30E-5	3.08E-5

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Portfolio-4-NSGA-II	0.6756	0.7280	0.7801	5.99E-4	7.89E-4	0.001	1.81E-5	2.22E-5	2.98E-5
Portfolio-4-RINSGA-II	0.8508	0.8517	0.8526	3.18E-4	4.26E-4	5.26E-4	2.79E-5	3.32E-5	4.05E-5
Portfolio-5-NSGA-II	0.7017	0.7459	0.7854	3.99E-4	8.06E-4	0.0016	7.90E-6	9.59E-6	1.85E-6
Portfolio-5-RINSGA-II	0.8710	0.8742	0.8751	2.65E-4	3.30E-4	5.00E-4	1.53E-5	2.02E-5	3.01E-5

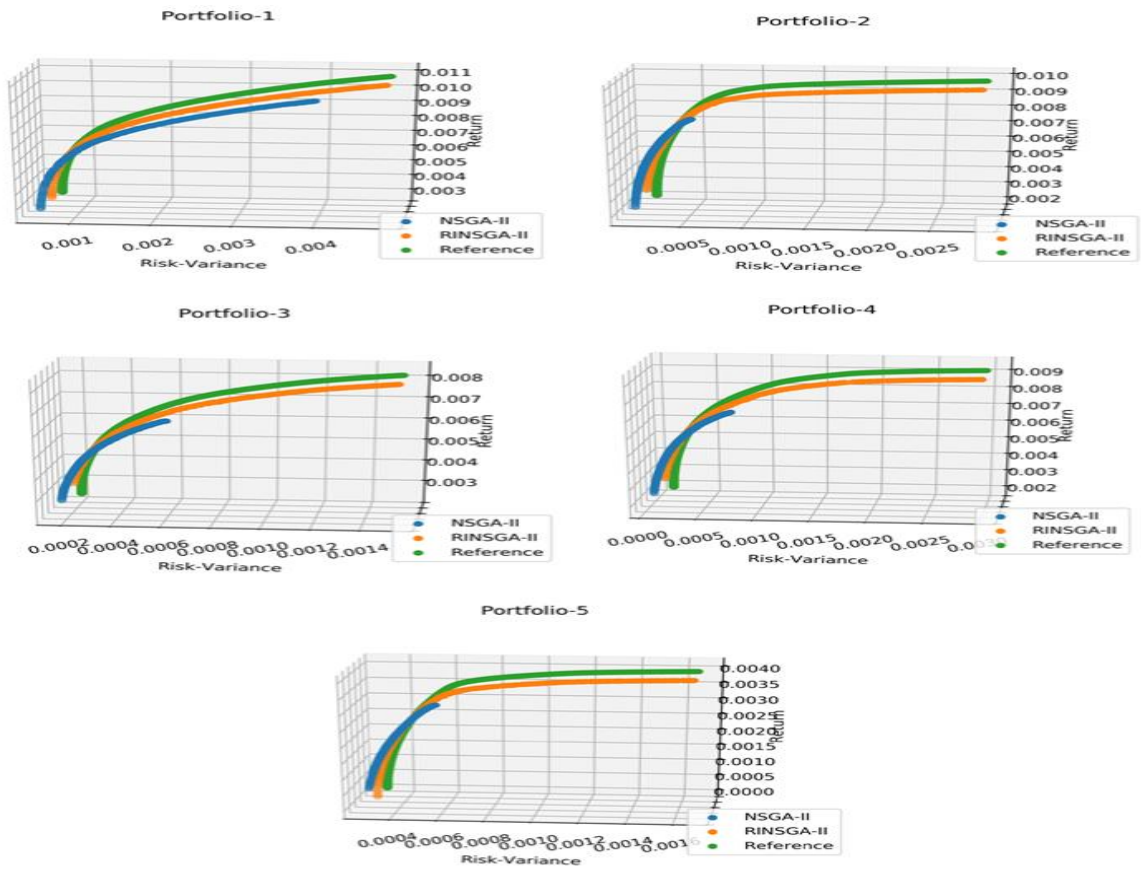

Figure 1: Pareto-front generated by NSGA-II and RINSGA-II

As indicated in the Table:5, the value of HV is more for RINGSA-II than NSGA-II. Hence, it shows that RINSGA-II produces consistent results compared to NSGA-II.

The independent-sample t-test conducted on HV values for RINGSA-II (*M=0.766, Var=1.27E-07*) and NSGA-II (*M=0.680, Var=0.000389*) indicates better convergence to Pareto-front. Even the values of GD for RINSGA-II was found lesser than NSGA-II. Though the difference in median values of NSAGA-II and RINSGA-II is marginal, RINSGA-II results are better. T-test, for GD values, is 16.11 significant with $p < 0.05$, thus supporting that the GD of RINGSA-II is less, indicates better convergence.

The values in the table above indicate that the solution spread is more in the case of RINGSA-II than to NSGA-II. The results indicate that the spacing of RINGSA-II is more, which means degradation of quality, particularly spacing measure is concerned. Figure:2 graphically presents the HV, GD, and spacing, and the Table reports the t-test results.

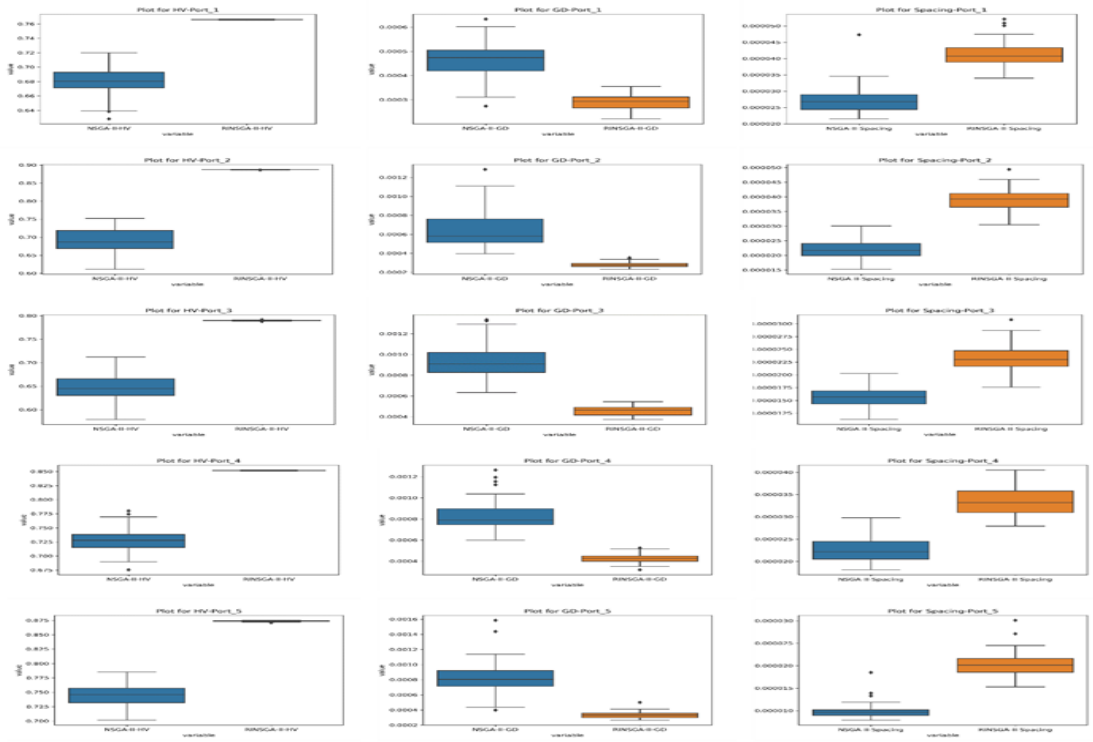

Figure 2: HV, GD, and Spacing

	Hypervolume								
	NSGA-II			RINSGA-II					
	M	Var	n	M	Var	n	T	df	p
Portfolio-1	0.6804	0.0004	50	0.7660	1.27E-07	50	30.65*	49	1.25E-33
Portfolio-2	0.6898	0.0011	50	0.8874	9.72E-08	50	42.11*	49	3.84E-40
Portfolio-3	0.6456	0.0007	50	0.7899	6.95E-07	50	39.40*	49	9.17E-39
Portfolio-4	0.7284	0.0005	50	0.8517	1.92E-07	50	38.75*	49	2.01E-38
Portfolio-5	0.7447	0.0003	50	0.8739	8.43E-07	50	52.98*	49	6.33E-45

* $p < .05$.

M = Mean Value

Var = variance

N = number of sample

	Generational Distance								
	NSGA-II			RINSGA-II					
	M	Var	n	M	Var	n	t	df	p
Portfolio-1	0.0005	4.82E-09	50	0.0003	1.15E-09	50	16.11*	71	2.07E-25
Portfolio-2	0.0006	3.33E-08	50	0.0003	7.4E-10	50	13.73*	51	9.30E-19
Portfolio-3	0.0009	2.71E-08	50	0.0005	2.11E-09	50	20.19*	57	1.21E-27
Portfolio-4	0.0008	2.06E-08	50	0.0004	1.78E-09	50	19.39*	57	8.89E-27
Portfolio-5	0.0008	4.62E-08	50	0.0003	1.88E-09	50	15.97*	53	6.57E-22

* $p < .05$.

	Spacing								
	NSGA-II			RINSGA-II					
	M	Var	n	M	Var	n	t	df	p
Portfolio-1	2.73E-05	1.72E-11	50	4.11E-05	1.71E-11	50	16.74*	98	1.67E-30
Portfolio-2	2.20E-05	9.67E-12	50	3.90E-05	1.38E-11	50	24.91*	95	1.97E-43
Portfolio-3	1.57E-05	4.59E-12	50	2.33E-05	6.18E-12	50	16.37*	96	1.57E-29
Portfolio-4	2.26E-05	8.33E-12	50	3.37E-05	1.10E-11	50	17.75*	96	4.33E-32
Portfolio-5	9.87E-06	3.12E-12	50	2.03E-05	6.76E-12	50	23.39*	86	4.84E-39

* $p < .05$.

1.2.1.1. MV model with Cardinality Constraints:

To determine the cardinality value, following the methods of Chiam et al. (2008), the graphs of risk against cardinality are plotted. It is evident that the risk gradually reduces with the increase in the cardinality value.

Figure:3 presents the risk against the cardinality of other portfolios.

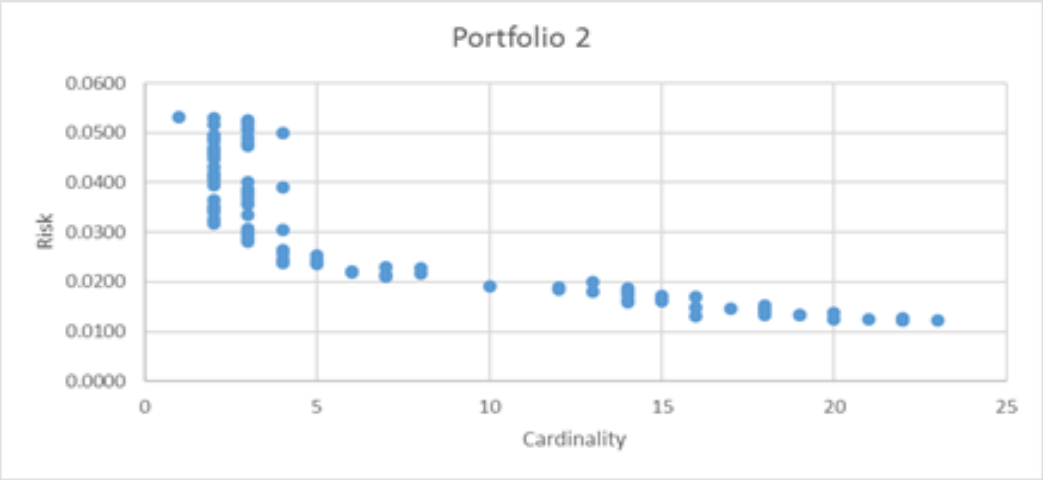

Figure 3: Cardinality vs. Risk for Portfolio 2

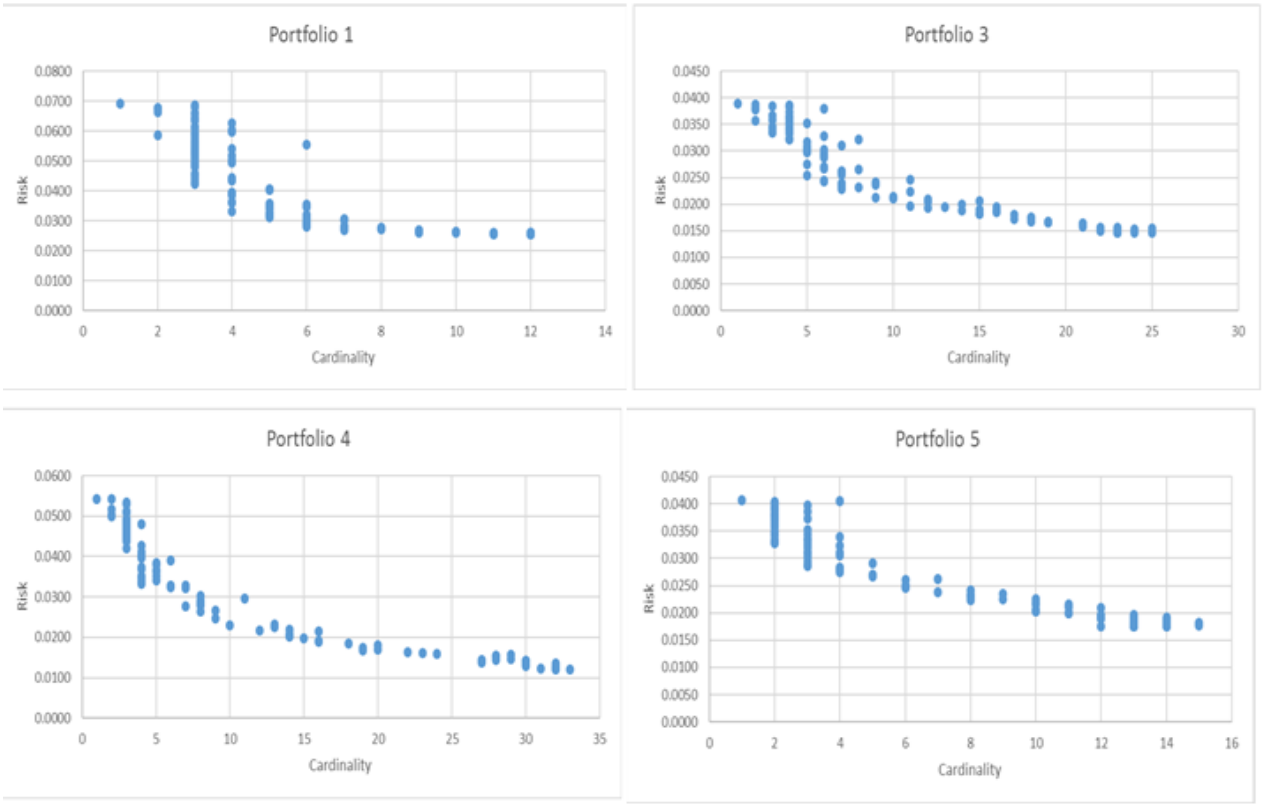

Figure 4: Cardinality vs. Risk for Portfolio 1,3,4, and 5

Table 6: Range of Cardinality

Portfolio	**Min**	**Max**
Portfolio-1	1	11
Portfolio-2	1	20
Portfolio-3	1	15
Portfolio-4	1	30
Portfolio-5	1	15

We impose the cardinality constraint on the MV model along with other parameters, and the results are present in Table:7.

Table 7: Comparison of NSGA-II and RINSGA-II for Cardinality Constrained MV Model

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Portfolio-1-NSGA-II	0.6662	0.7022	0.7643	1.17E-4	1.74E-4	3.613E-4	2.23E-5	3.03E-5	4.33E-4
Portfolio-1-RINSGA-II	0.7634	0.7643	0.7649	1.84E-4	2.58E-4	3.53E-4	3.26E-5	4.03E-5	5.08E-5
Portfolio-2-NSGA-II	0.6626	0.7222	0.7982	9.78E-5	2.92E-4	0.0013	1.87E-5	2.29E-5	3.19E-5
Portfolio-2-RINSGA-II	0.8818	0.8826	0.8831	1.72E-4	2.20E-4	3.17E-4	3.27E-5	3.99E-5	5.07E-5
Portfolio-3-NSGA-II	0.6139	0.6900	0.7362	1.47E-4	3.15E-4	5.46E-4	1.51E-5	1.78E-5	2.88E-5
Portfolio-3-RINSGA-II	0.7830	0.7848	0.7858	2.70E-4	3.69E-4	5.25E-4	1.85E-5	2.45E-5	3.00E-5
Portfolio-4- NSGA-II	0.6692	0.7221	0.7678	2.36E-4	5.39E-4	0.0012	1.94E-5	2.37E-5	3.17E-5
Portfolio-4-RINSGA-II	0.8436	0.8446	0.8454	2.28E-4	2.93E-4	3.77E-4	2.80E-5	3.39E-5	3.74E-5
Portfolio-5-NSGA-II	0.7279	0.7919	0.8483	1.17E-4	2.06E-4	5.07E-4	7.90E-6	1.17E-5	2.12E-5
Portfolio-5-RINSGA-II	0.8673	0.8705	0.8712	1.85E-4	2.61E-4	4.53E-4	1.55E-5	2.00E-5	2.99E-5

Note: The upper limit of the Cardinality value of each portfolio is as per Table 6 “Max” column

The results are described in Table-7. The value of HV is more for RINGSA-II as compare to NSGA-II. However, there is a marginal degradation of GD, in the case of Portfolio-5. For Portfolio-4, the GD of RINSGA-II is better than NSGA-II, and the spacing was found to be more in the case of RINGSA-II as expected. These results indicate that RINSGA-II can trace the larger front compare to the NSGA-II as far as the Portfolio Optimization problem is concerned, especially the high-risk portfolios.

Results Related to NIFTY 100 Index Using NSGA-II and RINSGA-II

We continue to explore the applicability of the RINGSA-II and NSGA-II algorithm for the Portfolio Optimization problem focused on equity included in the Nifty Old+Nifty New Index containing 100 equities (NIFTY100) and gradually impose the below constraints.

- Budget
- No short sales
- Round lot
- Cardinality
- Floor & Ceiling
- Sectoral
- Liquidity

We selected weekly equity price data of NIFTY100 equities, the Centre for Monitoring of Indian Economic(CMIE) PROWESS database spanning from the period of Jan'2015 to Dec'2018, for analysis and rolled over week by week for the optimization process. The range of cardinality used is 20- 25.

Table:8 below demonstrates the result.

Table 8: Performance Measure for NSGA-II and RINSGA-II

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
UC- NSGA-II	0.6801	0.7283	0.7604	3.26E-4	5.37E-4	9.43E-4	0.0176	0.0234	0.0820
UC-RINSGA-II	0.8194	0.8245	0.8281	3.02E-4	5.16E-4	8.21E-4	0.0706	0.0847	0.1538
Round Lot-NSGA-II	0.7114	0.7480	0.7865	2.36E-4	4.65E-4	0.0012	0.0161	0.0243	0.0574
Round Lot -RINSGA-II	0.8281	0.8322	0.8340	3.41E-4	4.89E-4	7.81E-4	0.0663	0.0815	0.1083

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Round Lot + Card-NSGA-II*	0.7482	0.7686	0.7751	3.63E-4	5.31E-4	0.0011	0.0194	0.0272	0.0383
Round Lot + Card-RINSGA-II*	0.7261	0.7504	0.7644	3.52E-4	6.31E-4	0.0022	0.0148	0.0191	0.0022
Round Lot + Crad + Floor-Ceiling-NSGA-II*	0.7352	0.7463	0.7551	3.04E-4	6.97E-4	0.0012	0.0098	0.0153	0.0243
Round Lot + Card + Floor-Ceiling-RINSGA-II*	0.7194	0.7383	0.7476	5.33E-4	9.64E-4	0.0015	0.0098	0.0149	0.0201

Algorithm	HV			GD			Spacing		
	Min	Med	Max	Min	Med	Max	Min	Med	Max
Round Lot + Card + Floor-Ceiling+ Sector-NSGA-II	0.7192	0.7327	0.7392	4.48E-4	6.92E-4	0.0015	0.0118	0.0142	0.0198
Round Lot + Card + Floor-Ceiling+ Sector-RINSGA-II	0.7064	0.7247	0.7339	6.77E-4	0.0010	0.0016	0.0086	0.0134	0.0173
Round Lot + Card + Floor-Ceiling+ Sector + volume-NSGA-II	0.7111	0.7339	0.7441	4.35E-4	7.35E-4	0.0015	0.0105	0.0153	0.0213
Round Lot + Card + Floor-Ceiling+ Sector + volume-RINSGA-II	0.6975	0.7290	0.7398	5.72E-4	9.85E-4	0.0023	0.0084	0.0143	0.0240

*Cardinality used is 20-25

We observe a few interesting facts. When we applied the NSGA-II and RINSGA-II algorithms on the Unconstrained model, i.e., the original MV model, as well as when we applied round lot constrained the performance of the RINSGA-II is better in respect to Hypervolume. The median values for round lot constrained problems are 0.7480 and 0.8322 for NSGA-II and RINSGA-II, respectively. However, when we applied the cardinality constraints of range 20-25, the HV values are 0.7686 and 0.7504, respectively. Therefore RINSGA-II algorithm is useful in tracing the high-risk area, but with the imposition of the cardinality constrained with high range, the algorithm is forced to be restricted in the low-risk area, and the performance of RINSGA-II degrades a little bit.

NIFTY 100 Index Using RINSGA-II

Using the RINSGA-II algorithm on the NIFTY100 datasets, we tested the impact of the constraints on the efficient front generated. The figures below depict the results:

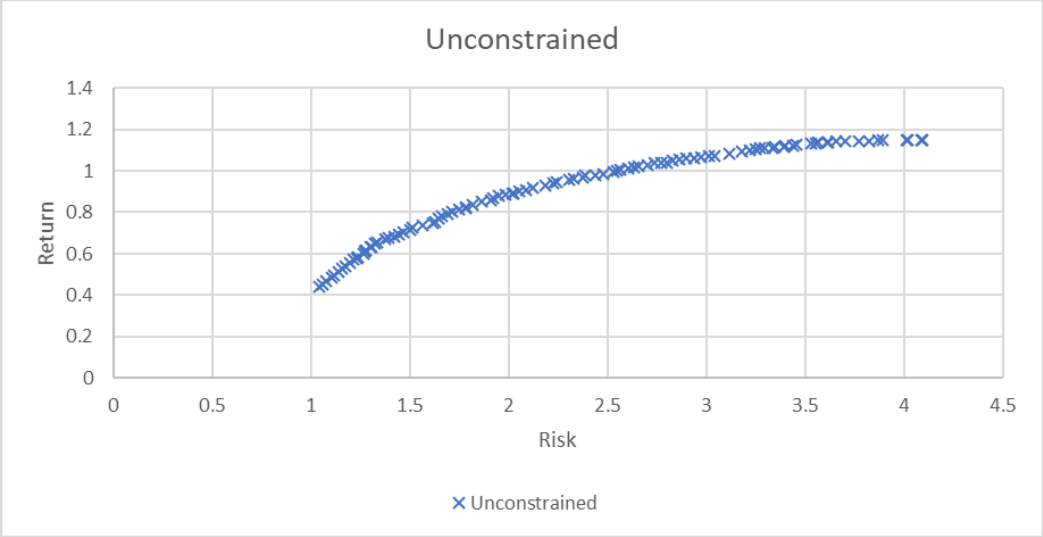

Figure 1: Unconstrained Pareto front

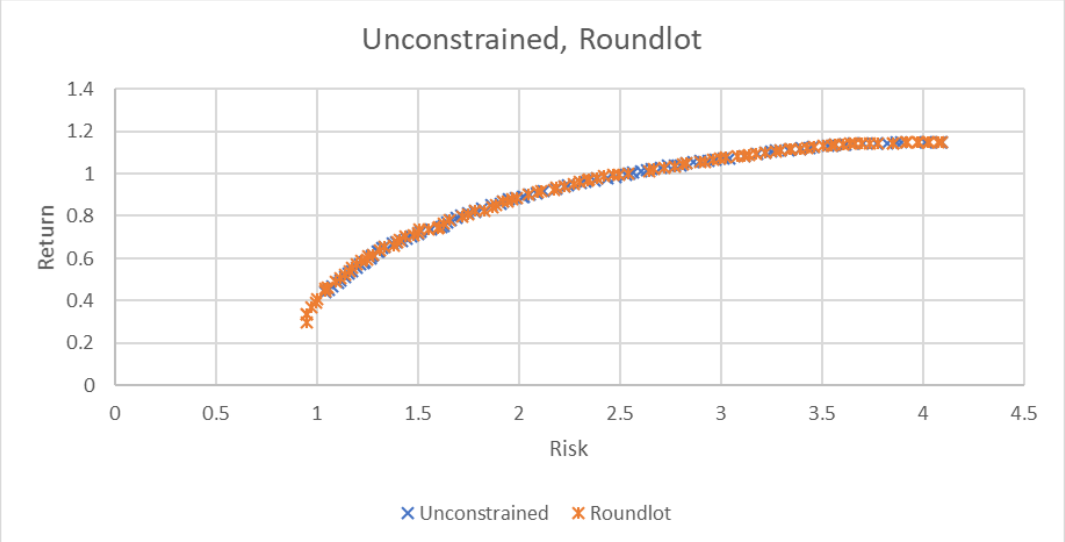

Figure 2: Unconstrained, Round Lot

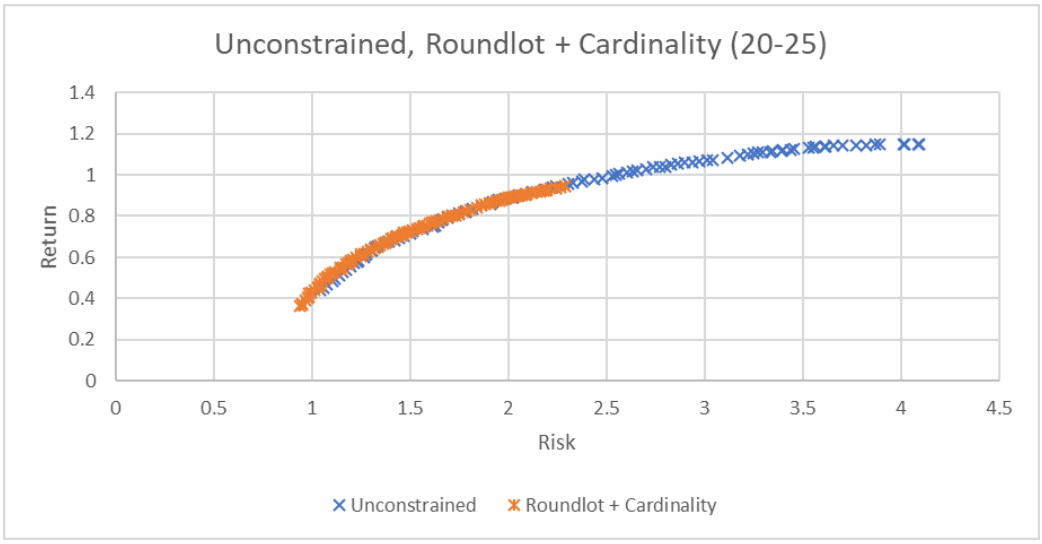

Figure 3: Unconstrained, Round Lot + Cardinality (20-25)

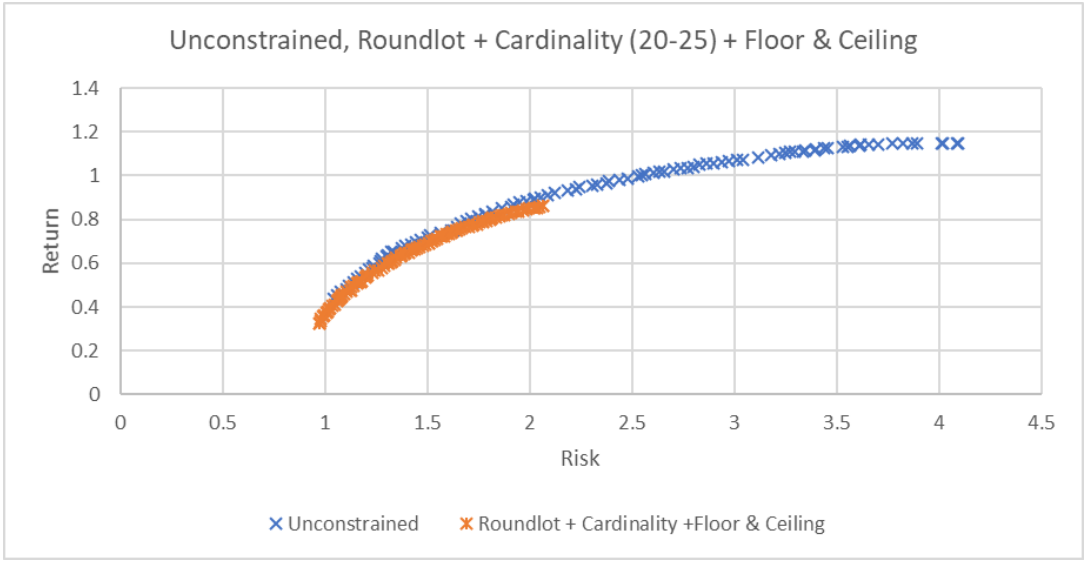

Figure 4 : Unconstrained, Round Lot + Cardinality + Floor & Ceiling

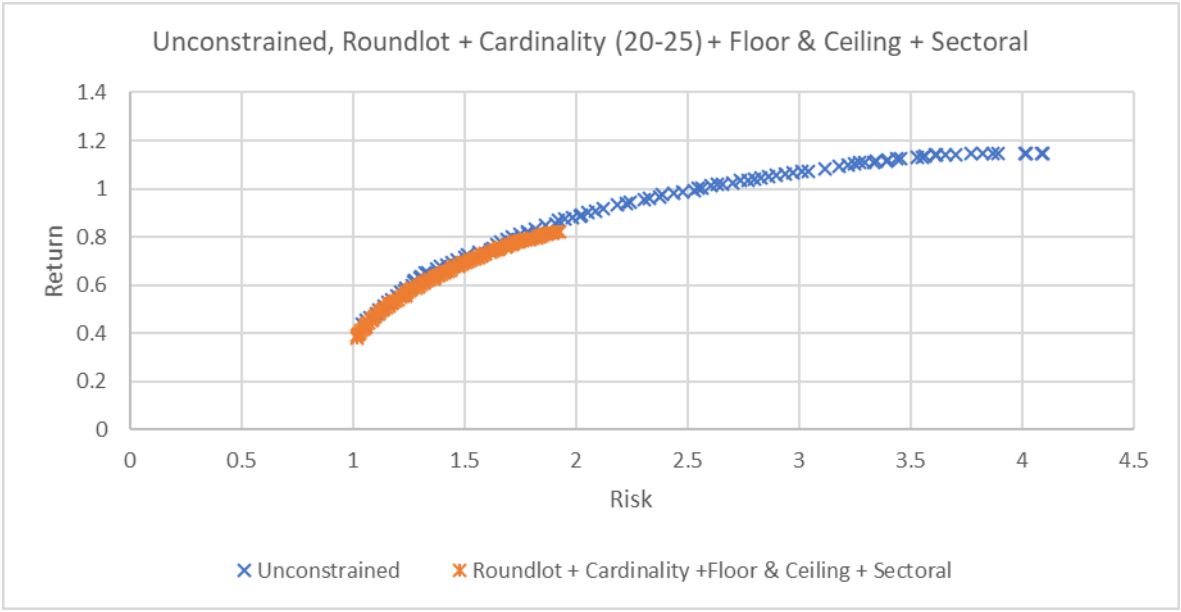

Figure 5 : Unconstrained, Round Lot + Cardinality + Floor & Ceiling + Sectoral

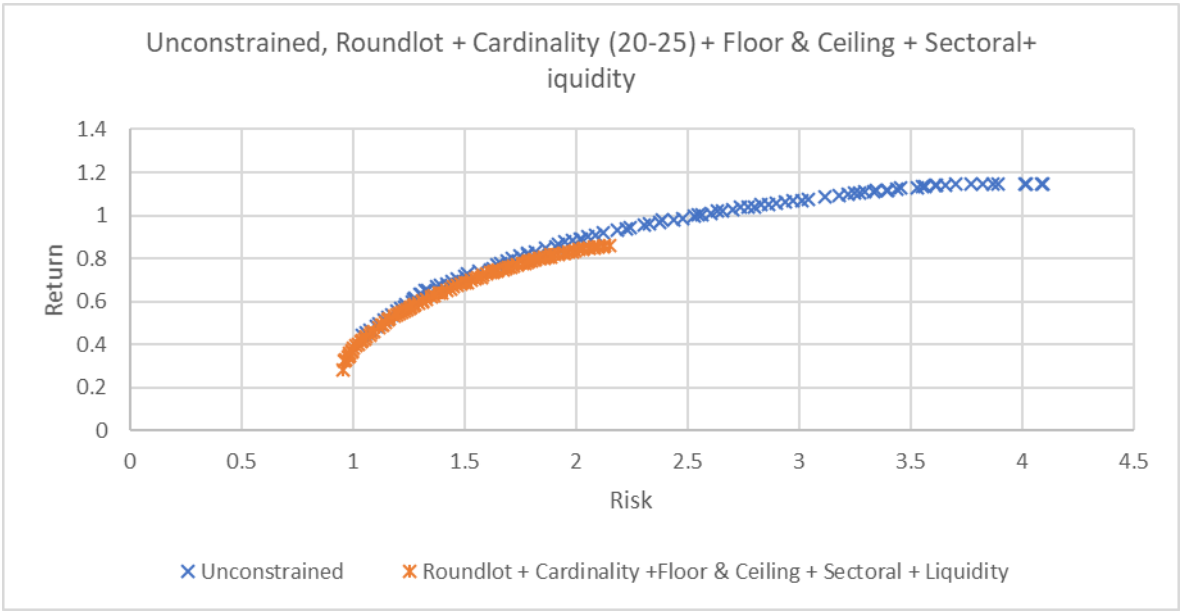

Figure 6 : Unconstrained, Round Lot + Cardinality + Floor & Ceiling + Sectoral + Liquidity

The figures indicate that the RINSGA-II can trace the efficient frontier even after the application of multiple constraints.

The imposition of the round-lot constraint does not impact the Pareto-efficient front. However, the imposition of the constraints, specifically cardinality, reduce the spread and constrain the portfolio generated at the lower-risk areas.

Out of sample test for the portfolios generated by RINSGA-II

For out-of-sample performance of the portfolios, we created portfolios every week using the price data on Friday and checked the risk-adjusted holding period return of the portfolios over one week, two weeks, four weeks (Month), 12 weeks (Quarter), 26 Weeks (six months) and compared the risk-adjusted returns against the Nifty-50 index for 52 weeks. The summarized view is as presented below:

Table 9: %of times the total number of portfolios beating Nifty return

One week	Two Weeks	One Month	One Quarter	Six Months
42.31	34.62	36.54	25.00	17.31

The result is as expected as the portfolios are formed based on the two parameters, like Risk and Return. It shows the promise that with the addition of other factors might produce better results.

1.3. Discussion

Form the experiments we have arrived at the following conclusions:

1) The RINSGA-II's results indicate that it is capable of tracking broader areas, including the high-risk area of the Pareto-front for the Portfolio Optimization problem.

2) The results obtained by the HPNSGA-II, are similar to those of GPNSGA-II NSGA-II and RINSGA-II when applied to Test problems. However, the performance degrades marginally when applied to the OR-library sets.
3) The application of round-lot constraints does not have any impact on the Pareto-front formation.
4) Application of other constraints, specifically Cardinality constraints, force the Pareto-front to be in the lower. Other constraints like Floor and Ceiling constraints, Sectoral constraint augment the same.
5) Out of the sample test suggests that 42.31% times more than 50% of the portfolios so formed beats the risk-adjusted Nifty return. So instead of random Adhoc investment, using the proposed algorithm to create optimized portfolios will have the potential to get a better return than the index in a short duration.

1.4. Contribution

This work contributes to knowledge in several ways.

1) We have proposed a new formulation, i.e., the HPNSGA-II algorithm, whose performances are comparable with the GPNSGA-II in case of test problems and better when tested against Portfolio optimization problem.
2) GPNSGA-II algorithm was tested against Test Problem, but the application against real-world application was lacking. In this work, we examined the HPNSGA-II and GPNSGA-II against the Portfolio Optimization problem for the first time.
3) RINSGA-II is a new type of formulation of the algorithm with different initialization process, in the static environment. This helps to trace the portfolios in the high-risk area.

4) We have proposed an enhanced formulation of the mean-variance model by the incorporation of several constraints which the investors face in reality.

1.5. Managerial Implication

The extended MV model proposed is much closer to reality than the original MV model and can be used by the Investors to determine the optimal allocations of the limited assets.

Optimized portfolios generated by RINSGA-II are capable to trace the larger portion of the efficient frontier. The result will help to cater to the high risk as well as low-risk investors.

Out of the sample test of the portfolio demonstrates that the portfolios created using enhanced mean-variance optimizers can beat the Nifty index in the short term. So, Portfolio Manager with better insight can use this algorithm to create portfolios, which will be able to beat the market index.

1.6. Future Direction

The model can be enhanced by the addition of factors like behavioral aspects of investors like loss aversion (Kahneman and Tversky, 1979.

As the list of constraints is not all-inclusive, the models can be enhanced by adding more constraints like Pre-Assignment constraints, Taxes-Transaction cost, market Capitalization constraints. The current model is a single period; however, there is a scope to enhance this as a multiperiod optimization problem.

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